## IV. "On the Intensity of Light reflected from certain Surfaces at nearly Perpendicular Incidence." By LORD RAYLEIGH, M.A., D.C.L., Sec. R.S. Received October 6, 1886.

[PLATE 3.]

In the present communication I propose to give an account of a photometric arrangement presenting some novel features, and of some results found by means of it for the reflecting power of glass and silver surfaces. My attention was drawn to the subject by an able paper of Professor Rood,* who, in giving some results of a photometric method, comments upon the lack of attention bestowed by experimentalists upon the verification, or otherwise, of Fresnel's formulæ for the reflection of light at the bounding surfaces of transparent media. It is true that polarimetric observations have been made of the *ratio* of the intensities with which the two polarised components are reflected; but even if we suppose (as is hardly the case) that these measurements are altogether confirmatory of Fresnel's formulæ, the question remains open as to whether the actual intensity of each component is adequately represented. This doubt would be set at rest, were it shown that Young's formulæ for perpendicular incidence (to which Fresnel's reduce), viz., $\left(\frac{\mu-1}{\mu+1}\right)^2$, agrees with experiment.

Professor Rood's observations relate to the effect of a plate of glass when interposed in the course of the light. He measures, in fact, the transmission of light by the plate, and not directly the reflection. No one is in a better position than myself for appreciating the advantages of this course from the point of view of experiment. In the first place, the incidence can easily be made strictly perpendicular, in which case no question arises of a separate treatment of the two polarised components of ordinary light. And, what is much more important, the interposition of the plate leaves the course of the light unchanged, and thus allows the alteration of intensity to be determined in an accurate manner with the simplest arrangements.

On the other hand, the measurement of the transmitted, instead of the reflected light, is open to grave objection on more than one ground. It may be doubted whether the influence of *absorption* is altogether negligible, even when the thickness of the plate is as small as that mentioned by Professor Rood, viz., 1·67 mm. But the feature which strikes me most unfavourably is the necessary magnification of error, when we deduce the proportion of light reflected

* 'Amer. Journ. Sci.,' vol. 49, 1870 (March); vol. 50, 1870 (July).

from the observed loss of light transmitted. The transmitted light is about 91 per cent.; and thus an error arising from the neglect of absorption, or from imperfect matches, amounting, say, to 1 per cent., leads to a relative error of more than 10 per cent. in the estimated reflection. The importance of this consideration may be illustrated by Professor Rood's actual results. In the first case recorded by him the observed transmission was 91·440 as against the theoretical 91·736. The difference 0·296 is indeed very small reckoned upon the transmitted light; but if we translate the results into terms of the reflected light, they present a different appearance. On the supposition that the whole loss in the transmitted light is due to reflection, we get for the intensity of the reflected light 8·560, which is to be compared with the theoretical 8·264. The difference 0·296 is now some $3\frac{1}{2}$ per cent., and is thus by no means insignificant. In the other case given by Professor Rood the discrepancy is greater still, amounting to 7 per cent. It may be remarked that in both cases the amount of the reflection appears to be in excess of that given by Young's formula. But the cause may lie in the assumption that the whole failure of transmission is due to reflection. And whatever the explanation may be, we can hardly agree with Professor Rood when he concludes that these experiments show "that the reflecting power of glass with the above index of refraction, *conforms in the closest manner to the predictions of theory*."

In the hope of being able to deal directly with the reflected light, I made a great many trials of various devices during the spring of 1885, but without finding anything satisfactory. Indeed, at one time, I had almost come round to the opinion that the difficulties of measuring the reflected light were so great that Professor Rood had shown a wise discretion in declining to face them, and that after all the best results would perhaps be reached through measurements of the transmitted light checked by the use of plates of different thicknesses so as to eliminate absorption.* If, indeed, we give up the perpendicular incidence, the objection founded upon the relatively small quantity of light reflected may be met; for at an incidence of 70°, about half the light (polarised in the plane of incidence) is reflected. In such an experiment it would of course be necessary to determine accurately the angle of incidence.

The difficulties referred to have their origin in the necessary alteration in the course of the light by the act of reflection. The direct and reflected light cannot be interchanged in any simple manner, and the shift necessary to bring about the substitution may easily lead to systematic error. In the apparatus (presently to be described) to which I was finally led, the difficulty seems to be fairly

* I was not aware until lately that Sir John Conroy was at work in this direction.

overcome so far as regards the accuracy of the results, but at the cost of several tiresome adjustments, impeding the ready trial and interchange of various reflectors.

My apparatus differs in several respects from that generally used for photometric purposes. Before describing it in detail, it may be worth while to indicate some of the considerations which led me to design it.

The photometers in ordinary use may be said to depend upon the principle of diffusion. If the illuminating candle, or lamp, be drawn back from the screen to double the original distance, the brightness of the screen as perceived by the eye is supposed to be quartered. This implies that (within certain limits) the brightness of the screen is independent of the apparent magnitude of the source of light (the total radiation being given), or that the light diffused by the screen in a particular direction (towards the eye) is independent of the direction of incidence. Reciprocally, the light incident in a definite direction is supposed to be diffused through a considerable angle with some approach to uniformity. There is no doubt that with proper arrangements this condition may be satisfied with sufficient accuracy for practical purposes. My object in formulating it is to show that the use of a diffusing screen in photometry is *necessarily* attended by an enormous reduction of light.

For our present purpose this loss of light is a serious matter. Weakened to $\frac{1}{25}$ by reflection from glass, the light of an ordinary candle or lamp is hardly sufficient to illuminate a diffusing screen properly, unless placed so close that measurement of the distances becomes uncertain. The difficulty might perhaps be got over by the use of incandescent electric lamps, but such were not at my command. When, as in Sir John Conroy's experiments,* the reflecting surfaces under test are metallic, or when (as above suggested) the observation relates to the transmission of light by an oblique plate of transparent material, the illumination given by a lamp may be adequate.

In my apparatus all the reflections are *regular*, and there is no further loss of light than the characters of the surfaces entail. An incidental advantage is that the accurate *flatness* of surfaces demanded by methods in which illumination is inferred from distance, is here unnecessary. The apparatus was first set up during the summer of 1885; but the glasses then at my disposal were not good enough, and when the parallel glass mirrors, &c., necessary for satisfactory working came into my possession, the season was so far advanced that I decided to postpone operations until the following summer.

* 'Roy. Soc. Proc.,' vol. 35, 1883, p. 26.

*Description of Apparatus.*

The light is admitted into the room through a pane of finely-ground glass fitted into the shutter. All other light is carefully excluded, and the walls and ceiling are blackened—an almost indispensable provision. The ground glass carefully cleaned* is illuminated not only by the direct light of the sky, but also by light from above reflected at a large mirror. The reception of light through a large angle not only favours the aggregate brightness and tends to moderate the changes due to passing clouds, but it makes the uniformity of the field more independent of the evenness with which the glass is ground. Under these circumstances, and when there is no sunshine, direct or reflected, falling upon the ground glass, the latter may be looked upon as a tolerably uniform source of diffused light. This uniformity, however, is not relied upon; but the arrangements are so made that the parts of the field compared are contiguous or identical, and are seen by rays which leave at the same angle. It will be convenient first to describe generally the course of the light, and afterwards the manner in which the adjustments were effected.

Proceeding from the ground glass (A), fig. 1, the light falls upon the transparent plates, B, B′, at which nearly equal parts are reflected. These plates are of worked glass about 6 inches by 4 inches, and are placed at the polarising angle. By this means are obtained two beams of polarised light of nearly equal, and of constant relative, brightness. The light transmitted by both plates is stopped at a screen, and takes no further part.

On the right hand side the light reflected at B is again reflected by a mirror of worked glass, silvered behind, at C, and assumes the direction CDF. D and F are alternative positions of the same mirror (also of worked glass, silvered behind). When the glass under test E is in use, the shifting mirror is in the position D, and the light follows the course CDEFH. In the contrary case, there is one reflection instead of two, and the ray takes the finally identical course CDFH. At H this central ray is reflected at the extreme edge of a speculum of silver-on-glass in the direction HI, to a small observing telescope, which is focussed upon this edge.

By adjustments that will presently be explained, it is secured that the reflections at D and F shall take place under the same angle, and therefore with the same (moderate) loss of light; but when E is in use the brightness is diminished some twenty times. To compensate this in the other position the disk G is introduced. It consists of a blackened disk of tin, from which (along a sufficient length measured radially) a sector is cut out, so that when the disk is

* Strong sulphuric acid is an excellent detergent for this purpose.

caused to revolve the view is cut off and blackness substituted for about nineteen-twentieths of the whole time. When the speed exceeds about twenty-five revolutions per second, there is no perceptible flickering, and the light is seen with a simple diminution of brightness. The idea of the method is so to adjust the angular opening that the effects of the glass under test and of the disk shall be equal.

The two brightnesses last considered can only be seen successively. They are separately tested with a comparison light, reflected at B′ from the same primary beam. After reflection at a silvered mirror, C′, and then at a blackened glass, D′, this light falls upon a silver-on-glass speculum at H′, and passes thence to the observing telescope at I. In setting up the apparatus a control over the brightness of the comparison light is obtained by varying the angle of incidence upon D′.

In order that the line of division between the two fields, as seen from I, may be quite sharp, it is necessary that the final reflector H be a speculum. To obtain a reflecting surface, perfect up to the very edge, a piece of silvered glass is carefully cut (on the glass side) with a diamond. If the operation is properly performed, the silver is left undisturbed, and when the plate is inclined, as at H, no part of the glass substratum is visible.

In adjusting the apparatus the object aimed at is to cause the central ray, ABB′, issuing from A on the ground glass, to assume ultimately the position HI, whether it proceed by the course on the left, or by either of the alternative courses on the right. As may be supposed, this is more easily said than done. All the reflectors require to be adjusted so as to be perpendicular to the plane in which the central ray is to travel. This plane is conveniently taken horizontal, so that every reflector has to be vertical.*

The central ray is defined by diaphragms at A, B, B′, points in the same horizontal straight line. At A the light is admitted through a small aperture only. At B, B′ the holes are cut in thin cardboard screens held in definite positions up to the glasses. It was found convenient to have them rather large—about half an inch in diameter. In setting up the apparatus the glasses B, B′, C are readily put into position, accuracy being required only in the levelling. The line CDF is now defined, and the next step is the more difficult one of fixing the two positions for the stand carrying the mirror D. This stand is (like all the others) provided with levelling screws, but these must not be used in passing from the one position to the other. A heavy

* The levelling of the reflectors was effected with the aid of the straight edge of a long board, adjusted until it coincided with the prolongation of its image. My assistant, Mr. Gordon, is expert at this adjustment of the edge to perpendicularity with the reflecting surface. The verticality of the latter is then tested by the application of a spirit-level to the edge of the board.

metal surface plate was laid down upon the table as the support for this stand, and carefully levelled. Under these circumstances the mirror if vertical in one position will remain vertical even though displaced; and this remains true, even though the feet of the stand do not rest immediately upon the plate, but upon small flat buttons of metal of uniform thickness, and perforated with equal holes, by which the feet of the stand are guided to definite positions. When the adjustments are complete, these buttons are fastened to the surface plate by dropping cement round their edges.

The position F may now be chosen at convenience, and without any particular care except in the levelling. The central ray, as fixed by the diaphragms, should fall near the middle of the surface. The other positions would also be somewhat arbitrary were it not for the necessity of securing the same angle of incidence and reflection in the two positions. To assist in this a small frame of brass wire is provided, carrying two pointers, and so arranged that it can always be placed in an absolutely definite position with respect to the mirror. By means of hooks it makes two contacts with the back, and two with the upper edge of the mirror. Of the other two contacts required to make up the necessary six, one is with the lower part of the face of the mirror, and the other with one of its vertical edges. Of the pointers, one (in the path of the incident ray) leads upwards, and the other (in the path of the reflected ray) leads downwards. By bending them suitably their extremities may be brought into the path of the central ray, so that when the eye is placed in such a position (H) as to see the central point A in the middle of the (apparently elliptical) aperture B,* this central point is just enclosed between the barely meeting pointers. By so choosing the second position, D, that this condition is again satisfied to an eye looking along ED, we secure not only the same angle of reflection but the use (for the central ray) of the same part of the glass. In making the adjustment we may first bring the pointer on which the incident ray strikes into the already determined line CD, and then rotating the apparatus about the vertical through this point, bring the second pointer to coincide with the reflected ray.

We have now to consider how to fix the position of E, the reflector under examination. Replacing the shifting mirror into its first position, F, we mark the line of the central reflected ray, FH, by needles, standing up from the table and as far apart as convenient. Transferring the shifting mirror to the position D, we have so to place E that the reflected ray shall coincide with the same line as before. For this purpose not only must the azimuth of E be correct, but its plane must be brought into the intersection of the already determined

* Auxiliary lighting, with a candle or otherwise, is sometimes necessary in order to see these apertures properly.

lines DE, FE. A levelled slab of glass is provided, on which to rest the feet of the stand carrying E. The mirror is now brought into a vertical plane, and may then be shifted on the slab without loss of this adjustment. The remaining double adjustment is best made systematically. By rotation about *any* vertical axis, the central ray may be caused to pass over one of the needles. If it fails to pass over the other, the axis of rotation must be shifted backwards and forwards until a suitable rotation allows satisfaction of both conditions. The ray now follows in both cases the course FGH, and the mirror H, with the sharp edge, may next be pushed in so as just to catch the ray in question and send it to the observing telescope (half of a small opera glass) at I.

The adjustments for the auxiliary light on the left hand side are a simpler matter. All the mirrors being levelled, the central ray is brought to the point H′, in the prolongation of IH. Nothing then remains but to turn the final (vertical) mirror round H′ until the reflected ray coincides with HI. When the eye looks in along this line, the bright spot should be seen in the same position from both mirrors.

To guard against accidental displacements, the movable pieces were usually secured with a little sealing wax. A diaphragm at K limits the field of view, and is so placed that the aperture is bisected by the division line H. It is not necessary to do more than allude to various screens employed to cut off stray light and render the room as dark as possible.

The principal trouble experienced, that of making and retaining the adjustments, is connected with the rather large scale of the apparatus, which made it difficult to use a single levelled bed for all the movable pieces. The question is thus suggested, what is it that fixes the absolute scale? And the rather unexpected answer must be—the diameter of the pupil of the eye, which is the only linear quantity concerned.*

In order to understand this it is necessary to bear in mind that although in describing the adjustments we speak of a single ray only, we are of necessity really dealing with a complete beam. The observation of a match requires that the two parts of the field of view have finite angular magnitudes, and from every point of the field there must proceed a pencil of rays limited by the pupil, or by the telescope. If all these rays are to be treated as sensibly parallel during their passage through the apparatus, certain limitations must be observed. For easy observation the field of view should subtend at the eye an angle of not less than a degree, so that if no telescope be employed the defect of parallelism must exceed this amount. The linear scale of the apparatus is not thus fixed, however, for we

* The wave-length of light may be regarded here as infinitely small.

might suppose the eye (armed when necessary with a focussing lens) to approach without limit the final mirrors. But if we do this we increase the defect of parallelism due to the aperture of the eye. It is true that we may elude the objection by contracting proportionally the effective aperture, but only at an expense of brightness, which cannot usually be afforded. In accordance with a universal rule, full brightness requires that the aperture of the eye be filled with light. In this way we see how it is that the aperture of the eye controls the size of the apparatus.

The employment of a telescope introduces a certain modification, which it may be worth while to state somewhat fully, as the principle is of general application. The extreme angle between the rays of the beam may be regarded as made up of two parts: (1) the angle subtended at the object-glass by the aperture in the diaphragm (K) near the final mirrors (upon which the telescope is focussed); (2) the angle subtended by the object-glass at the diaphragm. If—

$a$ = diameter of pupil,
$b$ = diameter of aperture in diaphragm,
$r$ = distance between telescope and diaphragm,
$m$ = magnifying power of telescope,
$\alpha$ = angular diameter of field of view presented to the eye,

then
$$\frac{mb}{r} = \alpha,$$

and the extreme angle between the rays of the beam—

$$= \frac{b}{r} + \frac{ma}{r} = \frac{\alpha}{m} + \frac{a}{b} \cdot \alpha.$$

We may here regard $\alpha$ and $a$ as given beforehand; and we see that with a given $b$ the first term may be reduced without limit by increasing $m$, and that then the defect of parallelism is proportional to $a$, the diameter of the pupil. If $m$ and $b$ can both be increased without limit, we may approach as nearly as we please to a state of things in which all the rays concerned are parallel. The preservation of full brightness throughout is already secured by the supposition that the effective aperture of the object-glass is $ma$.

The reasoning set forth above shows at any rate that the size of the apparatus cannot be reduced below a certain point, but I do not affirm that mine was not unnecessarily large. In addition to its other advantages, the use of a telescope gives facilities for obtaining a good focus upon the division line, an adjustment of great importance for the easy recognition of small differences of brightness.

The necessarily finite magnitude of the field of view involves a certain imperfection in this, and probably in other methods of photometry. We

can indeed secure that the lights seen in immediate juxtaposition come from the same part of the ground glass, but a corresponding perfection of adjustment does not apply to other parts of the field. If we suppose ourselves to be looking through the telescope at the ground glass, the part seen to the right of the division line really lies to the right on the ground glass. On the other side there is a distinction, according to the two positions of the shifting mirror. When the revolving disk is in use, the circumstances on the right hand side of the apparatus correspond to those on the left, and thus the part of the field seen to the left of the division line really comes from the left on the ground glass. The ground glass is thus seen much as if it were looked at directly, in spite of the separation of the light into two parts following distinct courses. On the other hand, when the additional reflector (under examination) is brought into play, there is another inversion, and the part of the ground glass seen to the left comes really from the right of the central line. In this case, therefore, it is the same part of the ground glass which is seen in both final mirrors. The distinction here pointed out would be of no consequence if the field were absolutely uniform, or if it were possible to compare the parts seen in immediate juxtaposition, without regard to the parts a little further removed. But if the original field vary slightly in brightness from right to left, it will be a question how far the eye would select for the match *continuity* of brightness across the division line, or how far it would demand equality in the *average* brightnesses of the two parts presented.

It now remains to describe certain accessories. During the observations it is necessary to have some means of varying the relative brightnesses of the two parts of the field without removing the mirrors or altering the width of the slit in the revolving disk. For this purpose a plate of glass (L), capable of rotation about a vertical axis, was introduced into the path of the light on the right hand side of the apparatus (between the second and third reflections). As the angle of incidence upon this plate increases, a greater proportion of the light is reflected and thrown away, and a less proportion is transmitted to the eye.

The observation consists in varying the azimuth of this plate until the match is satisfactory, after which the obliquity of the plate is measured. The transmission by the plate at the measured obliquity can then be found approximately from Fresnel's formula.* It may,

* A convenient table is given by Pickering ('Phil. Mag.,' vol. 47, 1874, p. 129). If A be the proportion of light reflected at a single surface, the transmission through a *transparent* plate is given by $(1-A)^2+(1-A)^2A^2+(1-A)^2A^4+ \ldots\ldots = (1-A)/(1+A)$. The whole reflection is thus $2A/(1+A)$, from which Pickering's table is calculated. An erratum may be noted. For 65° the reflection should be 39·6, not 38·4, the value of A being supposed to be $\sin^2(\theta-\theta_1)/\sin^2(\theta+\theta_1)$, while $\sin\theta = 1{\cdot}55 \sin\theta_1$. There are some other minor inaccuracies.

perhaps, be objected that the use of this formula assumes the very thing that the experiments were principally intended to test; but the objection is evaded, almost if not altogether, when the aperture in the disk is so nearly adjusted to the ideal width that the oblique plate comes to take nearly the same azimuth for both sets of readings, *i.e.*, with and without the use of the mirror under examination. The use of the formula to allow for a small outstanding difference of obliquities can lead to no appreciable error. If on a first trial a large difference be found, a corrected aperture is calculated with the aid of Pickering's table, and the disk readjusted or replaced.

A fixed oblique plate has sometimes been used on one or other side of the apparatus in order to effect a rough adjustment of the brightness, and to bring the necessary obliquity of the rotating plate to a convenient amount (30°—60°). This was less trouble than a readjustment of the mirrors on the left, with an alteration in the angle of incidence upon the black glass D′.

In taking an observation the adjustment of the relative brightnesses was facilitated by a device which may now be described. If the attempt be made to secure an absolute match between the two parts of the field in view, a doubt is apt to arise as to whether the disappearance of the division line is due to the success of the adjustment or to fatigue of the eyes, leading, as in my case it very rapidly does, to imperfect focussing. This difficulty is less felt when the adjustment is under the immediate control of the observer, who can then satisfy himself of the sensitiveness of his eye by making the necessary displacement; but in the present experiments (on account of the distance of the telescope) it was convenient to employ an assistant. A glass plate, perpendicular to the path of the light, and attached to a sort of pendulum, was therefore provided on the left hand side, in such a manner that by pulling and letting go a string it could be introduced or withdrawn at pleasure. The effect of the plate would be to stop some 8 per cent. of the light, and the adjustment was so made that with glass *in* the (apparent) right of the field was as much too dark as it was too bright when the glass was *out*. The difference of brightness, amounting according to the above estimate to 4 per cent., was always fully apparent, and probably no setting more than 2 per cent. in error would be allowed to pass, giving, as such would do, a difference of 6 per cent. on the one side and of 2 per cent. upon the other.* Since the auxiliary light is eventually eliminated, it makes no difference, of course, whether we take for the comparison the full light, or the mean of the lights with and without the interposition of the plate.

A 2 per cent. error in single settings may lead to a 4 per cent.

* The accuracy of the settings falls much short of that attained by Professor Rood.

error in the comparison of the effects of the reflector and of the disk; and accordingly (since this may occur in either direction) an 8 per cent. discrepancy in the results is *possible*. This, however, would be very unlikely, and with a two- or three-fold repetition of the individual settings would be practically out of the question.

The revolving disks, used to diminish the light on the right hand side of the apparatus in about the same degree as by the mirror under test, were cut from tin plate, about 9 inches in diameter, and carefully centred. The angular apertures were finally calculated from measurements of the chord of the arc, and of the radius. It is important that the disks be thoroughly blackened, in view of the assumption that no light reaches the eye except during the passage of the aperture. Here is one reason why it is desirable to keep the room as dark as possible. The disk should also be properly balanced. On one occasion a curious and at first puzzling effect was observed. The division line, which should present no visible width, sensibly widened, appearing sometimes darker than the nearly balanced adjoining fields, and sometimes, though more rarely, appearing relatively bright. The explanation is to be found in a vibration of the mirror, whose edge forms the division line, in a horizontal direction perpendicular to the line of sight, the vibration being communicated from the revolving wheel through the floor to the table upon which the mirrors stood. It is evident that if the two lights under comparison were equal, not merely on the average, but at every moment of time, such a movement of the mirror would have no disturbing influence, and could not make the division line visible. But it is otherwise when one of the lights is intermittent, and the vibrations of the mirror are executed (as here they must be) in the same period. For suppose that at the moment when the division line is advanced, so as to invade still further the field from the back mirror, the light is reaching the eye through the aperture in the disk. In this case the parts near the edge of the vibrating mirror will be sending to the eye the full light due to this part of the field. During the remainder of the vibration, no light should reach the eye, but if this mirror retreats, the back mirror sends its continuous light from the same apparent place, so that when the angular opening in the disk is small, it is possible for the part of the field over which the division line vibrates to present an almost doubled brightness, combining in fact the illumination of the two parts of the field. A different phase relation may evidently lead to an abnormal diminution of brightness in the same region. These effects disappeared when the disk was better balanced.

*Prism of Crown Glass* (I).

In ordering a glass for the purpose of determining the reflecting power of a surface, a prism was preferred to a plate, both on account of the easier separation of the reflections from the front and back surface, and also because the refractive index could be determined more readily. During the observations the hind surface was coated with black varnish, the effect of which, however, in annulling the second reflection, was far from complete.

With this glass, carefully cleaned (but not repolished), six sets of observations were made, four by myself and two by Mrs. Sidgwick. Each set consisted of three or four settings with the glass in operation, and about the same number with substitution of the revolving disk. The following is a set of readings by myself on August 7th, 1886:—

Face of Prism (I).

| | Reflection. | | Revolving disk. |
|---|---|---|---|
| | 39°·0 | ...... | 40°·3 |
| | 38·2 | ...... | 42·7 |
| | 37·8 | ...... | 43·0 |
| | 39·1 | ...... | 42·0 |
| | — | ...... | 43·5 |
| Mean.. | 38·5 | ...... | 42·3 |

The angles here given are the obliquities of the adjustable glass plate used to graduate the intensity. According to Pickering's table, calculated from Fresnel's formula ($\mu = 1·55$), the effect of this plate at 38·5° would be to reflect 15·7 per cent. of the light incident upon it. The light transmitted is therefore 84·3 per cent. In like manner the light transmitted by the plate at an obliquity of 42·3° is 82·5 per cent.

In order to complete the calculation of the reflecting power of the glass surface, we must know the proportion of light transmitted by the revolving disk. Measurements gave for the chord of aperture of this disk in fiftieths of an inch 45·0, corresponding to a radius of 174·25. The angle of aperture is thus 14° 50′ = 14·83°. Accordingly the factor expressing the reduction of light by use of the disk is—

$$14·83/360 = 0·04119.$$

The reflection from the face of the glass prism is thus—

$$\frac{82·5}{84·3} \times 0·04119 = 0·0403.$$

The following is a summary of the results obtained at this time:—

Face of Prism (I).

| Lord Rayleigh. | | | Mrs. Sidgwick. | |
|---|---|---|---|---|
| Aug. 4.... | 0·0411 | ...... | Aug. 4.... | 0·0405 |
| „ 5.... | 0·0413 | ...... | „ 5.... | 0·0413 |
| „ 7.... | 0·0403 | ...... | | — |
| „ 9.... | 0·0413 | ...... | | — |
| Mean.. | 0·0410 | ...... | Mean.. | 0·0409 |

Final mean...... 0·04095.

Since this number is very nearly the same as that (0·04119) due to the disk alone, we see that the result is scarcely at all dependent upon the correctness of the assumed effect of the oblique plate.

It now remains to make comparison with the reflection as given by Fresnel's formula, viz.:—

$$\frac{\sin^2(\theta-\theta')}{\sin^2(\theta+\theta')}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A)}$$

where $\sin\theta/\sin\theta'=\mu$, and $\mu$ is the refractive index.

The index was determined in the usual way from the angle of the prism ($i$) and from the minimum deviation (D). The value of $i$ was found to be 9° 50′. The minimum deviation of soda light on one side was 5° 5′, and on the other 5° 4½′. Thus D = 5° 4¾′, and

$$\mu = \frac{\sin\frac{1}{2}(\mathrm{D}+i)}{\sin\frac{1}{2}i} = 1{\cdot}5141.$$

The angle of incidence ($\theta$), which was the same in all the observations with the various reflectors, was measured by determining the angle ($2\theta$) between the incident and reflected ray. This measurement does not require great precision, for a change of a whole degree in the value of $\theta$ would alter the reflection about 1 per cent. only. I found—

$$\theta = 13° \ 52'.$$

With these values of $\theta$ and $\mu$, we find—

$$\frac{\sin^2(\theta-\theta')}{\sin^2(\theta+\theta')} = 0{\cdot}04514,$$

about 10 per cent. in excess of the reflection actually observed.

In order to satisfy myself that the deficiency of reflection was real and permanent, this prism was remounted after a thorough cleaning, and further observations were taken, as summarised in the following table. The revolving disk was the same as before:—

Prism of Crown Glass (I), Remounted.

| | Lord Rayleigh. | | Mr. Gordon. | | Mean. |
|---|---|---|---|---|---|
| Aug. 24 ....... | 0·04085 | .... | 0·04100 | .... | 0·0409 |
| „ 25, morn... | 0·04183 | .... | 0·04199 | .... | 0·0419 |
| „ 25, even.*. | 0·03950 | .... | 0·04030 | .... | 0·0399 |
| „ 26 ....... | 0·04190 | .... | 0·04170 | .... | 0·0418 |
| Mean.... | 0·04102 | .... | 0·04125 | .... | 0·04113 |

The difference between 0·04113 and the mean previously found, viz., 0·04095, has no significance.

In consequence of the detection of a greatly augmented reflection from another glass surface (II, below), as the result of a repolish with putty powder, this surface also was submitted to similar treatment. Immediately afterwards, on August 30th, a much increased reflection was observed, the numbers by two observers being—

0·0481, 0·0472; mean, 0·0476.

The disk gave, as before, a transmission 0·04119; so that the numbers for the repolished face depend too much upon the assumed effect of various obliquities of the inclined plate to be fully trustworthy, even were they sufficiently numerous to guard against accidental errors. But they proved, unequivocally, a considerable increase in reflecting power as the result of the repolish.

In view of these results, a new disk was prepared of angular aperture about $17\frac{1}{4}°$, and, consequently, with a transmission equal to 0·04763. The numbers obtained with this are shown in the following table:—

Prisms of Ground Glass (I) Repolished.

| | Lord Rayleigh. | | Mr. Gordon. |
|---|---|---|---|
| Aug. 30, aft......... | 0·0461 | .... | 0·0451 |
| „ 31, morn....... | 0·0451 | .... | 0·0454 |
| „ 31, aft......... | 0·0448 | .... | 0·0447 |
| Mean ...... | 0·0453 | .... | 0·0451 |

Final mean = 0·0452.

The observed result now agrees remarkably well with that calculated from Fresnel's formula; but unfortunately it depends more (for about 5 per cent. of its value) than could be wished upon the use of the oblique plate.

* The considerable discrepancy shown in this set of readings was probably caused by insufficiency of light.

### *Prism of Crown Glass* (II).

So soon as it appeared that the reflection from the face of prism (I) fell so much short of what was to be expected in accordance with Fresnel's theory, I tried another prism whose surface was still older than that of (I). The event proved a still more marked deficiency. With the aid of the disk giving transmission 0·04119, the following numbers were obtained:—

Prism (II), before Repolishing.

| | Lord Rayleigh. | | Mrs. Sidgwick. | | Mr. Gordon. |
|---|---|---|---|---|---|
| Aug. 26.. | 0·0349 | .... | — | .... | 0·0344 |
| ,, 27.. | 0·0342 | .... | 0·0350 | .... | — |

Mean.. 0·0346.

Although somewhat dependent upon the assumed effect of the oblique plate, this number is far too low to be consistent with anything obtainable from Fresnel's formula with an admissible index.* This circumstance suggested a repolishing of the surface, which, however, was superior to that of (I), so far as could be judged from close inspection in a favourable light. The repolishing was executed by Mr. Gordon by means of a disk of wood charged with putty powder and mounted in the lathe. Observation now demonstrated a remarkable improvement in the reflecting power, as the following numbers will show:—

Prism (II), after Repolishing.

| | Mrs. Sidgwick. | | Lord Rayleigh. | | Mr. Gordon. |
|---|---|---|---|---|---|
| Aug. 28, morn... | 0·0491 | .... | 0·0488 | .... | — |
| ,, 28, aft..... | — | .... | 0·0479 | .... | 0·0473 |
| ,, 30, morn... | — | .... | 0·0484 | .... | 0·0481 |

Mean.. 0·0483.

Here again too much depends upon the oblique plate, the transmission of the disk being only 0·04119; but there can be no doubt of an increase in the reflection of something like 30 per cent. If we may argue from the number obtained from prism (I) after repolishing,

* If $e^2$ be the proportions of light reflected at incidence $\theta$, then Fresnel's formula is equivalent to—

$$\tan\theta_1 = \frac{1-e}{1+e}\tan\theta,$$

by which $\theta_1$ is found. The index is then given by $\sin\theta/\sin\theta_1$. In the present case

$$e^2 = 0{\cdot}0346, \quad e = 0{\cdot}1860, \quad (1-e)/(1+e) = 0{\cdot}08140/1{\cdot}1860,$$

so that, since $\theta = 13° \, 52'$, $\qquad \theta_1 = 9° \, 37'$, $\quad \mu = 1{\cdot}434$.

under nearly similar circumstances, viz., 0·0476, we may conclude that the true reflecting power of this prism is about 0·0460.

Altogether the evidence favours the conclusion that recently polished glass surfaces have a reflecting power differing not more than 1 or 2 per cent. from that given by Fresnel's formula; but that after some months or years the reflection may fall off from 10 to 30 per cent., and that without any apparent tarnish.

The question as to the cause of the falling off, I am not in a position to answer satisfactorily. Anything like a disintegration of the surface might be expected to reveal itself on close inspection, but nothing of this kind could be detected. A superficial layer of lower index, formed under atmospheric influence, even though no thicker than $\frac{1}{100000}$ inch, would explain a diminished reflection. Possibly a combined examination of the lights reflected and transmitted by glass surfaces in various conditions would lead to a better understanding of the matter. If the superficial film act by diffusion or absorption, the transmitted light might be expected to fall off. On the other hand, the mere interposition of a transparent layer of intermediate index would entail as great an *increase* in the transmitted as falling off in the reflected light. There is evidently room here for much further investigation, but I must content myself with making these suggestions.

*Plate Glass Silvered Behind.*

This glass was silvered chemically by the milk-sugar process, and by transmitted light showed the sky of a normal deep blue colour. The film was not polished. In determining the efficiency of this and other good reflectors, the black glass mirror D′ was replaced by one silvered behind. The first trial without a revolving disk gave for the reflecting power 0·82. This result, of course, depended entirely upon the assumed influence of various obliquities of the adjusting plate. A disk was therefore prepared with two opposite projecting teeth, in which the ratio of aperture to circumference turned out on careful measurement to be 0·8230. This number, therefore, represents the transmission of light by the disk. Using this disk I found the following values for the reflecting power of the mirror for light incident upon it at an angle of 13° 52′:—

| | |
|---|---|
| Aug. 11.............. | 0·823 |
| „ 12.............. | 0·833 |
| Mean...... | 0·828 |

This result relates, like all the others, to light polarised in the plane of incidence. Mirrors of this kind are durable, and not being exposed to tarnish are more convenient than specula, whenever the double reflection is not objectionable. The high reflecting power is a satisfactory feature.

*Silver-on-Glass Speculum.*

This was the silver side of the same glass as the last, polished with wash leather and a little rouge. The milky film was not perfectly removed. Four observations, not over concordant, probably in consequence of variation of reflecting power at different parts of the surface, gave—

| Lord Rayleigh. | | Mrs. Sidgwick. | | Mr. Gordon. |
|---|---|---|---|---|
| Aug. 14.. 0·902 | .... | Aug. 16.. 0·933 | .... | Aug. 14.. 0·920 |
| ,, 16.. 0·895 | | | | |

Mean.. 0·912.

The surface was then repolished and remounted with the following results:—

| Lord Rayleigh. | | | Mr. Gordon. | |
|---|---|---|---|---|
| Aug. 18.... | 0·950 | ...... | Aug. 18.... | 0·952 |
| ,, 19.... | 0·938 | ...... | ,, 19.... | 0·911 |
| | — | ...... | ,, 21.... | 0·938 |
| Mean.... | 0·944 | ...... | Mean.... | 0·934 |

Mean.. 0·939.

The increase in efficiency may have been due to a more careful selection of the best polished central part as much as to actual improvement in the polish of the speculum as a whole. The transmission of the disk used with this surface is 0·9105.

Sir John Conroy* found an even higher number (0·975) as the reflecting power of silver films for light polarised in the plane of reflection, and incident at 30°.

*Mirror of Black Glass.*

A plate of opaque glass has the advantage that the influence of the hinder surface is eliminated without more ado; but, on the other hand, it lends itself less readily to determinations of index. The following results were obtained with such a plate:—

| | Mrs. Sidgwick. | | Lord Rayleigh. |
|---|---|---|---|
| July 29.... | 0·0580 | ...... | — |
| ,, 30.... | 0·0581 | ...... | 0·0570 |
| ,, 31.... | 0·0583 | ...... | 0·0572 |
| Aug. 2..... | 0·0574 | ...... | 0·0578 |
| ,, 3..... | 0·0581 | ...... | 0·0577 |
| Mean ... | 0·0580 | ...... | 0·0574 |

Final mean.. 0·0577.

* 'Roy. Soc. Proc.,' vol. 37, 1884, p. 38.

During these observations a disk was employed giving transmission 0·0577, so that in this case the final result is absolutely independent of the effect of the adjustable oblique plate. It will be observed that the separate results obtained by Mrs. Sidgwick and by myself differ, even in the means, by 1 per cent. This is not the only instance in which the errors have presented a suspiciously systematic appearance; but the differences being always small could not be submitted to any satisfactory examination. It rarely happened, for instance, that Mrs. Sidgwick and I could find definite fault with each other's settings.

When these results were first obtained, I thought that they would turn out to be too high for agreement with Fresnel's formula, supposing that the index of the glass was low. A subsequent measurement of the specific gravity, however, gave reason for suspecting that the glass might be flint, a conclusion confirmed by determinations of the refractive index.

These were made by two methods: (1) by observation of the polarising angle in air, (2) by observation of the angle at which total reflection sets in when the mirror is immersed in bisulphide of carbon. The first is, perhaps, the simpler in respect of experimental arrangements, but it is open to the objection that the inference of the refractive index from the polarising angle is somewhat theoretical.

The black glass was mounted upon the turntable of an ordinary goniometer. In the focus of the collimator was placed a wire, seen dark in a bright field of view. Various positions of the turntable were then tried, such that on rotating a Nicol held at the eye the dark patch appeared to pass somewhat to the right or to the left of the collimator wire. After each observation the web of the telescope was set to coincidence with the collimator wire, and a reading taken. Success depends in some degree upon the use of a suitable light. Sunshine diffused through ground glass answered the purpose very well.

| Right. | | Left. | | Central. |
|---|---|---|---|---|
| 64° 25′ | .... | 63° 56′ | .... | 64° 7′ |
| 16 | .... | 40 | .... | 64 0 |
| 17 | .... | 55 | .... | — |
| 14 | .... | 55 | .... | — |

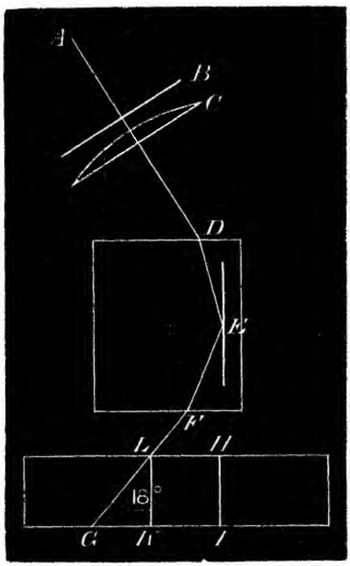
The table gives a set of circle readings. In the first column the patch was to the right of the collimator wire, in the second to the left, and in the third there was no appreciable deviation. We may, therefore, take as the reading for the polarising angle 64° 5′, with a probable error not exceeding 3′ or 4′. The reading for a direct setting of the telescope upon the collimator wire was $-5'$, so that the polarising angle is $\frac{1}{2}(180-64° \ 10')=57° \ 55'$. Whence according to Brewster's law—

$$\mu = \tan 57^\circ\ 55' = 1{\cdot}5952.$$

This relates to white light.

To find the index of refraction by the method of total reflection, the mirror was mounted vertically in a small tank of plate glass, cemented with glue and treacle, and containing bisulphide of carbon. The mirror E, as shown in fig. 2, was parallel to one of the sides of

FIG. 2.




the tank, and a cover was provided to check evaporation. A uniform field of homogeneous light could be obtained from a salted spirit lamp, A, with the aid of a plate of ground glass, B, and a collimating lens, C. The eye looking in along such directions as GF, is able to mark with considerable accuracy the direction in which total reflection begins. By the aid of plumb-lines, &c., this direction and that of the face of the mirror (seen from above) were marked upon a board, and it appeared that the angle GLK, between the face of the mirror and the direction GLF of the first totally reflected emergent light was 18°.

A beautiful variation in the experiment may be made by replacing the spirit lamp with a candle, and subsequently analysing the reflected light by a direct vision prism. For this purpose a screen carrying a slit should be interposed as near F as conveniently may be. As the incidence of the light upon the black glass becomes more grazing total reflection sets in, but first at the violet end of the spectrum.

When the eye is looking nearly in the right direction, the spectrum appears to be covered by a veil proceeding from the red end up to a point dependent upon the precise direction of the light. By slightly shifting the eye, the veil may be made to reach any desired part of the spectrum, and then we know for what ray total reflection is just commencing. By bringing the veil to touch the soda line (rendered visible with the aid of the spirit lamp), precisely the same direction was found as had previously been marked out with use of homogeneous light. It would be possible in this way to determine with considerable accuracy the dispersive powers of opaque bodies.

The angle of 18°, being measured in air, is not the complement of the true angle of reflection. If we take 1·630 as the index of $CS_2$ for soda light, we find for this angle

$$\sin^{-1}\left(\frac{\sin 18^\circ}{1\cdot630}\right)=10^\circ\ 56'\ ;$$

whence for the index of the glass relative to soda light,

$$\mu=1\cdot630\cos 10^\circ\ 56'=1\cdot600.$$

The amount of reflection according to Fresnel's formula, with an incidence of 13° 52′ and an index 1·600, is 0·05726, a little *less* than that actually observed. The agreement is as good as could be expected, but it should be noticed that this mirror was merely cleaned and not repolished with putty powder. If repolishing were to produce as much effect in this case as upon the acute-angled prism (I), Fresnel's formula would be left considerably in arrear.*

P.S. Nov. 9, 1886.—I am indebted to Mr. Glazebrook for a determination of the refractive index of the prism of crown glass II. He finds $\mu = 1\cdot5328$. The introduction of this into Fresnel's formula ($\theta = 13^\circ\ 52'$) gives for the reflecting power 0·0477.

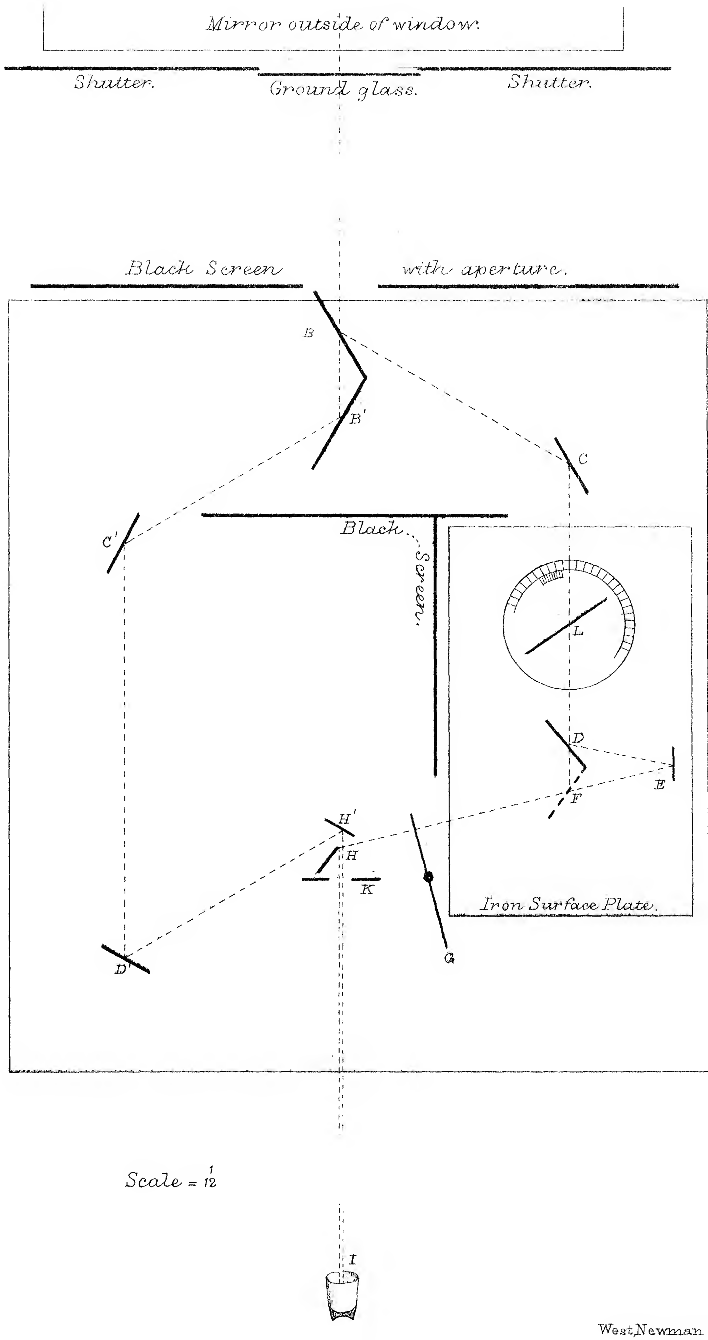
## V. "A Theory of Voltaic Action." By J. Brown. Communicated by Lord Rayleigh, Sec. R.S. Received October 4, 1886.

[Plates 4 and 5.]

1. From a series of experiments made more or less continuously during the last five years the following conclusions have been drawn:—

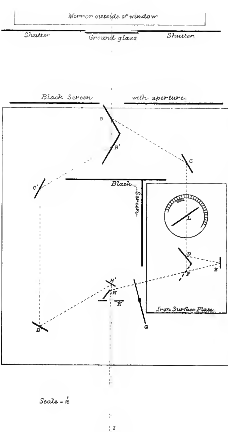
That the difference of potential near two metals in contact as observed either by the bi-metallic condenser (Volta's) method,

* Some of the results here given were communicated to the British Association at Birmingham, where also was read a paper by Sir John Conroy on the same subject.




West, Newman & Co. sc.

FIG. 2.



Mirror outside of window
Shutter
Ground glass
Shutter
Black Screen
with aperture
B
B'
C
C'
Black Screen.
L
D
E
F
H'
H
K
G
Iron Surface Plate
D'
Scale = 1/12
I